Levels of Depth of Knowledge for Mathematics

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, or straight algorithmic procedure should be included at this lowest level.

Some examples that represent but do not constitute all of Level 1performance are:

•Count to 100 by ones and by tens.

•Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8).

•Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.

•Enter measurement data into a data table.

•Identify the variables indicated in a two-dimensional graph.

Level 2 (Basic Application of Concepts & Skills) includes the engagement of some mental processing beyond a habitual response. A Level 2 standard or assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2.

Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common and require more mental processing.

Some examples that represent but do not constitute all of Level 2performance are:

•Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

•Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end.

•Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l)

•Apply properties of operations as strategies to add and subtract rational numbers.

•Measure and record data and produce graphs of relevant variables.

•Graph proportional relationships, interpreting the unit rate as the slope of the graph.

Level 3 (Strategic Thinking & Complex Reasoning) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. However, an activity that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3.

Some examples that represent but do not constitute all of Level 3performance are:

•Explain why addition and subtraction strategies work, using place value and the properties of operations.

•Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

•Given a real-world situation, formulate a problem.

•Organize, represent, and interpret data obtained through experiments or observations.

•Formulate a mathematical model to describe a complex phenomenon

•Justify a solution to a problem.

•Analyze a deductive argument.

Level 4 (Extended Thinking &Complex Reasoning) in mathematics involves the application of level three processes and skills over an extended period. This is likely to incorporate demands from other content areas (e.g., English language arts, science), in the development and support of mathematical arguments that describe some real-world phenomenon or situation.

Some examples that represent but do not constitute all of Level 4 performance are:

•Derive a mathematical model to explain a complex phenomenon or make a prediction.

•Complete a project requiring the formulation of questions, devising a plan, collecting data, analyzing the data, and preparing a written report describing the justification of the conclusions reached.